GEOB 300 - Reading Package Lectures 4 and 5

REVIEW - RADIATION BASICS

Electromagnetic waves. Radiation can be described as a form of energy due to the rapid oscillations of electromagnetic fields. The oscillations may be considered as travelling waves characterized by their wavelength λ (distance between successive wavecrests). In most atmospheric applications we are concerned with wavelengths in the approximate range 0.1 to 100 μm (1 μm = 10^{-6} m, see Reading Package 1), representing only a very small portion of the total electromagnetic spectrum (Figure 1, coloured in blue and red). The visible portion of the spectrum, to which the human eye is sensitive, is an even smaller fraction extending from ~0.4 μm (violet) to ~0.7 μm (red). Radiation is able to travel in a vacuum, and all radiation moves at the speed of light ($c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$), and in a straight path. Hence, instead of wavelength, we can express radiation by its frequency ν (in oscillations per second, i.e. $\mathrm{s^{-1}}$):

$$\nu = \frac{c}{\lambda} \qquad \star \qquad (2.1)$$

Particle properties. Radiation can be also described by discrete bundles of energy that have properties similar to particles. Quantum physics tells us that radiation is transferred as discrete packets called quanta (or *photons* if in the visible part of the spectrum). The wavelength is uniquely related to the particle energy of a quantum so that it is possible to calculate the energy flux at any given wavelength or waveband. The energy of a quantum is equal to Plancks constant ($h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$) multiplied by the frequency:

$$e = h\nu \qquad \star \qquad (2.2)$$

Example 1: What is the energy content of a photon of yellow light (0.55 μm)? First calculate the frequency ν of the yellow-light radiation:

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8\,\mathrm{m\,s^{-1}}}{0.55 \times 10^{-6}\,\mathrm{m}} = 5.5 \times 10^{14}\,\mathrm{s^{-1}}$$

Then use Equation 2.2:

$$e = h\nu = 6.63 \times 10^{-34}\,\mathrm{J\,s} \times 5.5 \times 10^{14}\,\mathrm{s^{-1}} = \underline{3.6 \times 10^{-19}\,\mathrm{J}}$$

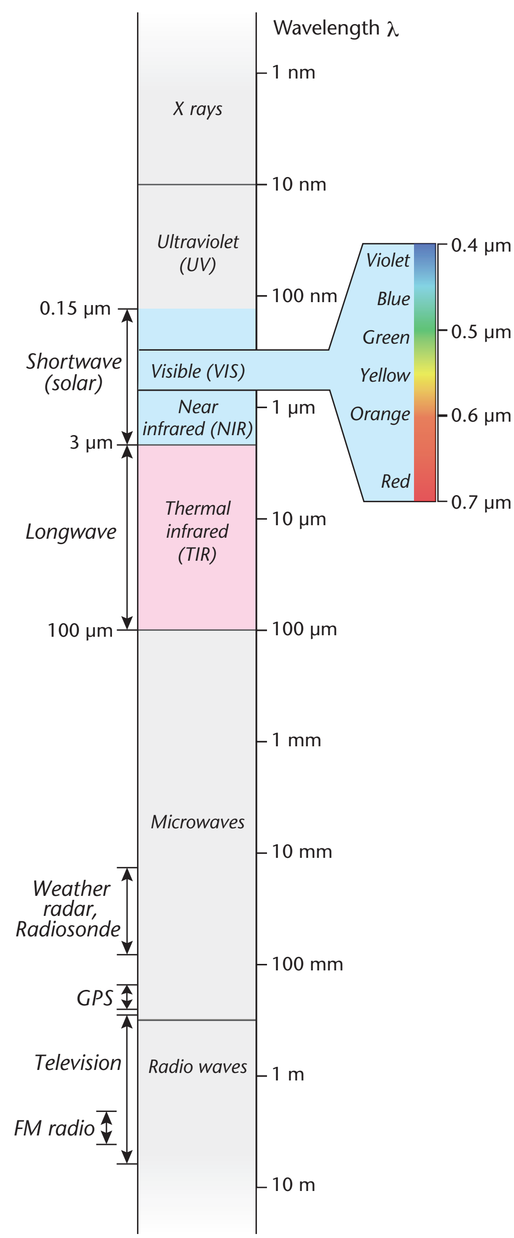

Figure 1: The electromagnetic spectrum.

To avoid very small numbers we can refer to the energy in 1 mole of photons (multiply by Avogadros number $N = 6.023 \times 10^{23}\,\mathrm{mol}^{-1}$). For example, a photon of blue light ($\lambda = 0.4\mu\mathrm{m}$) would have the energy $2.99 \times 10^5\,\mathrm{J\,mol^{-1}}$. This approach to radiant energy is widely used in plant sciences and ecology since in the process of photosynthesis, plants respond to the number of photons rather than simply to the total energy absorbed. Further, the radiative flux can be directly related to chemical reactions if expressed in moles. Plants can only use photons in the visible range of the spectrum which is also called *photosynthetically active radiation* (*PAR*, 0.4 - 0.7 μm). We can quantify radiation in the PAR range as *photosynthetic photon flux density* (*PPFD*) and express it in $\mathrm{mol\,m^{-2}\,s^{-1}}$

Example 2: In the PAR band, the median wavelength is $0.51\,\mu\mathrm{m}$. *At midday when the solar radiation in the PAR waveband is* $500\,\mathrm{W\,m^{-2}}$, *what is the PPFD?*

First, we calculate the energy of a photon at $0.51\,\mu\mathrm{m}$ following the calculation in Example 1:

$$\begin{aligned} e = h\,\frac{c}{\lambda} &= 6.63 \times 10^{-34}\,\mathrm{J\,s} \times \frac{3 \times 10^8\,\mathrm{m\,s^{-1}}}{0.51 \times 10^{-6}\,\mathrm{m}} \\ &= 3.9 \times 10^{-19}\,\mathrm{J} \end{aligned}$$

Converted to one mole of photons:

$$\begin{aligned} E = N\,e &= 3.9 \times 10^{-19}\,\mathrm{J} \times 6.02 \times 10^{23}\,\mathrm{mol^{-1}} \\ &= 2.35 \times 10^5\,\mathrm{J\,mol^{-1}} \end{aligned}$$

This is related to an energy flux density of $500\,\mathrm{W\,m^{-2}} (= 500\,\mathrm{J\,m^{-2}\,s^{-1}})$

$$\begin{aligned} PPFD &= \frac{500\,\mathrm{J\,m^{-2}\,s^{-1}}}{2.35 \times 10^5\,\mathrm{J\,mol^{-1}}} \\ &= \underline{2.1 \times 10^{-3}\,\mathrm{mol\,m^{-2}\,s^{-1}}} \end{aligned}$$

We see that with 500 $\mathrm{W\,m^{-2}}$ irradiance in the PAR band, the photosynthetic photon flux density is about $2130\,\mu\mathrm{mol\,m^{-2}\,s^{-1}}$. The result means that at the maximum, photosynthesis can convert $2130\,\mu\mathrm{mol}\,CO_2$ per second to carbohydrates (per square metre of optimally oriented leaf surface). As we will see later, plants are less efficient, and the actual rate of photosynthesis is much smaller. The calculation considers the spectrum to be continuous over the PAR waveband.

Radiation Laws. All bodies possessing energy (i.e. whose temperatures are above absolute zero, 0 K = -273.2°C) emit radiation. If a body at a given temperature emits the maximum possible amount of radiation per unit of its surface area in unit time then it is called a *black body*. Such a body has a surface emissivity ε equal to unity.

Less efficient radiators have emissivities between zero and unity. The relation between the amount of radiation emitted by a black body, and the wavelength of that radiation at a given temperature is given by *Plancks Law*. In graphical form this law shows the spectral distribution of radiation from a full radiator to be a characteristic curve (Figure 2). The shape consists of a single peak of emission at one wavelength ($\lambda_{\max}$), and a tailing-off at increasingly higher and lower wavelengths. The form is so characteristic that in Figure 2 the same shape of a Planck curve, describes the emission spectra from full radiators at different temperatures (Earth, Sun). However, the total amount of radiation given out and its spectral composition are different as it can be seen by the corresponding axes (left and right).

Emittance is the radiant flux density emitted by any surface. The emittance of each body E_b is proportional to the area under the curve (coloured in Fig 2, including the tail at longer wavelengths that has been truncated). This is the basis of the *Stefan-Boltzmann Law*:

$$E_b = \sigma T^4 \tag{2.3}$$

where, σ is Stefan's constant ($5.67 \times 10^{-8}\,\mathrm{Wm^{-2}\,K^{-4}}$, and T is the surface temperature of the body (K). In the typical range of temperatures encountered in the Earth-Atmosphere (E-A) system (-15 to 45°C) a change of 1 K in T of a full radiator results in a change of the emitted radiation of between 4 and 7 W $\mathrm{m^{-2}}$. If the body is not a full radiator, equation 2.3 must be re-written to include the value of the surface emissivity ε:

$$E_g = \varepsilon \sigma T^4 \qquad \star \tag{2.4}$$

Note that the energy emission given by these equations is the radiant flux density ($\mathrm{J\,s^{-1}}$ = W) from a unit area ($\mathrm{m^2}$) of a plane surface into the overlying hemisphere.

The effect of temperature change on the wavelength composition of the emitted radiation is embodied in *Wiens Displacement Law*. It states that a rise in the temperature of a body not only increases the total radiant output, but also increases the proportion of shorter wavelengths of which it is composed. Thus as the temperature of a full radiator increases, the Planck curve is progressively shifted to the left, and the wavelength of peak emission ($\lambda_{\max}$) moves with it so that:

$$\lambda_{\max} = \frac{b}{T} \qquad \star \tag{2.5}$$

with $\lambda_{\max}$ in metres and T on the Kelvin scale.

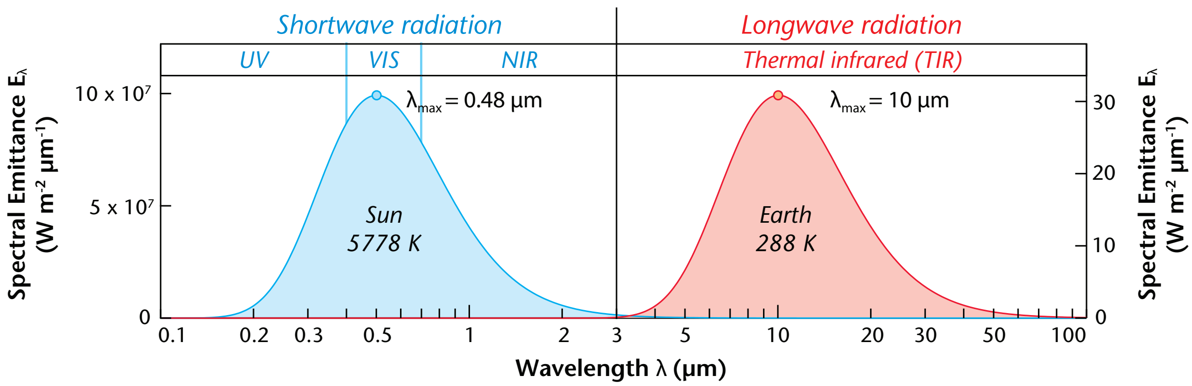

Figure 2: Planck curves for back body radiators at temperatures of 5778 K (Sun's surface temperature, with left vertical axis), and 288 K (Earth's surface temperature, right-hand vertical axis). $\lambda_{\max}$ is the wavelength at which energy output per unit wavelength is maximal.

Shortwave and longwave radiation. From equation 2.5 and in Figure 2 it can be seen that the Sun's peak wavelength is about $0.48\,\mu$m (in the middle of the visible spectrum), whereas for the E-A system it is about $10\,\mu$m. Typical wavelengths for radiation from the Sun extend from 0.15μm (ultra-violet) to about $3.0\,\mu$m (near infra-red), whereas E-A system radiant wavelengths extend from $3.0\,\mu$m to about $100\,\mu$m, well into the infrared. In fact the difference between the two radiation regimes is conveniently distinct; about 99% of the total energy emitted by the two systems (Sun, Earth) lies within these limits. On this basis atmospheric scientists have designated the radiation observed in the range $0.15 - 3.0\,\mu$m to be *shortwave* or solar radiation, and that in the range $3.0 - 100\,\mu$m to be *longwave* or thermal infrared radiation.

RADIATION GEOMETRY

Sun-Earth geometry. Figure 3 illustrates the geometrical relations between the Earth and the solar beam radiation, and defines the following angles:

ϕ *Latitude* of the location, the angle between the equatorial plane and the site of interest (point X in the figure), considered positive in the Northern and negative in the Southern Hemisphere.

δ - *Solar declination*, the angle between the Suns rays and Earth's equatorial plane.

Z *Solar zenith angle*, the angle between the Suns rays and the local zenith direction. The zenith direction in the figure is the extension of the line connecting the centre of the Earth and the point X, i.e. directly overhead. The complementary angle $(90° - Z)$ is β, the solar altitude or elevation (not shown), and is the angle between the Sun and the local horizontal.

h The *hour angle* is the angle through which the Earth must turn to bring the meridian of the site X directly under the Sun. It is a function of the time of day. It is related to, but distinct from, Ω.

Ω The *solar azimuth angle*, which is the angle between the projections onto the horizontal plane of the site of both the Sun and the direction of geographic north. The azimuth angle is measured clockwise from north (0360°).

Spherical trigonometry gives the following relationships between the angles:

$$\begin{aligned} \cos Z &= \sin\phi\sin\delta + \cos\phi\cos\delta\ \cos h \\ &= \sin\beta \end{aligned} \tag{2.6}$$

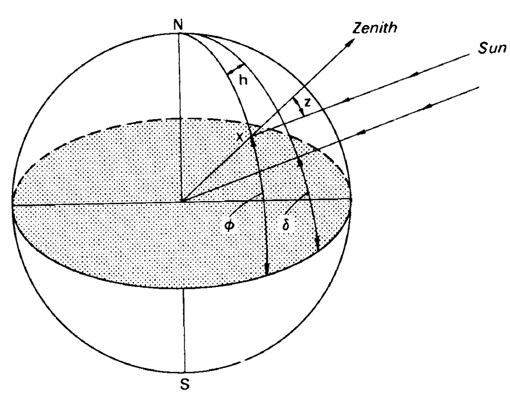

Figure 3: Geometrical relations between the Earth and the solar beam (S). The angles (see text) are defined with reference to the equatorial plane (shaded) and the point of interest (X).

and for the solar azimuth angle we have to separate by time of day

$$\begin{aligned}\cos\Omega &= \frac{\sin\delta\cos\phi - \cos\delta\sin\phi\cos h}{\sin Z} \quad (2.7)\\ &\text{for t} < 12\\ &= 360^\circ - \frac{\sin\delta\cos\phi - \cos\delta\sin\phi\cos h}{\sin Z}\\ &\text{for t} > 12\end{aligned}$$

where t is *local apparent solar time* (LAT) using hours in a 24-hour clock. The solar declination δ is dependent only upon the day of the year; to a first approximation it may be calculated from:

$$\delta \approx -23.4^\circ \cos\left(360^\circ \frac{DOY + 10}{365}\right) \quad (2.8)$$

where *DOY* is the number of the day in the year. A more accurate formula can be found in the lecture notes.

Since 1 hour is equivalent to 15° of Earth rotation the hour angle h is given by:

$$h = 15(12 - t) \quad (2.9)$$

where t is again *LAT*. How to calculate *LAT*? (i) add (subtract) 4 min to local standard time for each degree of longitude the site is east (west) of the standard meridian of the given time zone; this gives the *local mean solar time* (*LMST*). Now: (ii) algebraically subtract the result from the *equation of time* (see Lecture notes, $LAT = LMST - \Delta_{\mathrm{LAT}}$) to (i) in order to get the local apparent solar time.

Using the cosine law of illumination and equation 2.6 it is straightforward to calculate the solar irradiance impinging at the top of the atmosphere (Extraterrestrial irradiance K_{Ex}) at any location and time:

$$K_{Ex} = I_0 \left(\frac{R_{av}}{R}\right)^2 \cos Z \quad (2.10)$$

where I_0 is the solar constant ($1366.5\,\mathrm{W\,m^{-2}}$), R_{av} is the average distance of Earth and Sun, and R is the current distance between Earth and Sun (given by the time of the year, see Lecture 4, slide 20 for calculation).

It is also useful to note that the daylength t_d (number of hours with the Sun above the horizon) for an unobstructed horizontal site can be found by solving equation 2.6 for $Z = 90^\circ$ (or $\beta = 0^\circ$) to give the value of h at sunrise and sunset (h_{ss}). Since the Suns path is symmetric about solar noon (LAT), at which time $h = 0^\circ$, it follows that $t_d = 2\,h_{ss}/15$. Also since the diurnal course of direct beam irradiance (S) is approximately sinusoidal in the absence of cloud the value at any time, t, can be approximated by:

$$S = S_{\max} \sin\left(\frac{\pi\, t}{t_d}\right) \quad (2.11)$$

where $S_{\max}$ is the value of the maximum irradiance at solar noon.

Sun path diagrams. The calculation of the position of the Sun in the sky for a given location and time can be achieved using equations 2.6 and 2.7. By hand the procedure is somewhat laborious; by computer it is fairly simple, but there is also a useful way of both obtaining values of solar altitude and azimuth and of gaining a visual appreciation of the situation. Figure 4a schematically shows the paths taken by the Sun across the sky vault at the times of the summer solstice (21 June), the equinoxes (21 September and 21 March) and the winter solstice (21 December), relative to an observation site on the surface.

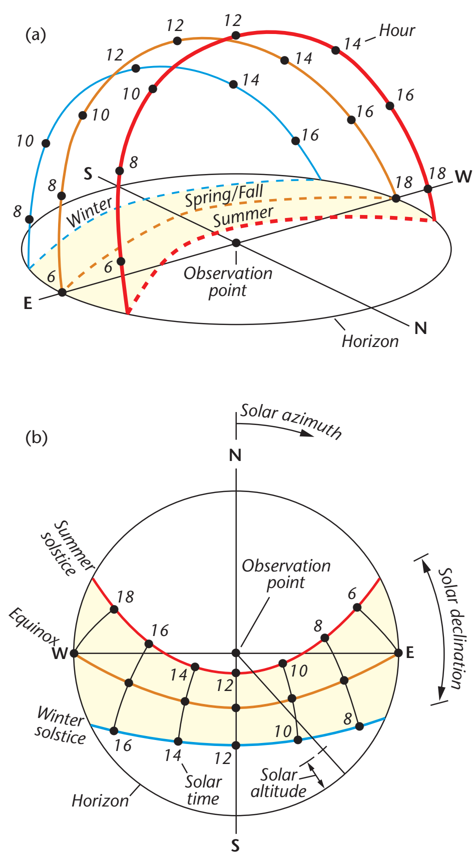

Figure 4: (a) Paths of the Sun across the sky vault for a mid- latitude location in the Northern Hemisphere at the times of the solstices and equinoxes, (b) The projection of (a) onto a two- dimensional plane thereby forming the basis of a Sun path diagram.

The positions at 2 h intervals of solar time are noted by small dots on the paths. Figure 4b shows the projection of the three-dimensional arcs onto a two-dimensional plane. This is the basic framework of a Sun path diagram.

In a sun path diagram the outer limit of the circle represents the horizon, and the centre is the observation site. The projection is equidistant, so that solar altitudes are equally spaced. The heavy curves, running from side to side, are the Sun paths at different times of the year (i.e. different solar declinations). The heavy curves, running up-and- down, divide the paths into solar time of day. On the course webpage[1] you will find a Sun path diagrams for any latitude. To locate the Sun for a specific location and time: select the chart appropriate to the latitude, the solar declination appropriate to the date, and using the solar time to fix the position of the Sun on the diagram, read off the solar altitude and azimuth.

RADIATION TRANSMISSION

Radiation of wavelength λ incident upon a substance must either be *transmitted* through it or be *reflected* from its surface, or be *absorbed*. This is a statement of the conservation of energy. By expressing the proportions transmitted, reflected and absorbed as ratios of the incident energy, we define the transmissivity (Ψ_λ), the reflectivity (α_λ) and the absorptivity (ζ_λ) and it follows from the energy conservation that:

$$\Psi_\lambda + \alpha_\lambda + \zeta_\lambda = 1 \qquad \star \qquad (2.12)$$

The Atmosphere depletes the solar beam as it passes through by an amount depending upon the length of its path, and the properties of the air. Gases, droplets and particles (especially ozone, water vapour, dust, clouds and smoke) have their own set of radiative properties with respect to the incident short-wave radiation, thus part of the beam is reflected (scattered), a part is absorbed and the rest is transmitted to the surface. The ratio of the extraterrestrial input to these amounts defines the bulk atmospheric reflectivity, absorptivity and transmissivity (α_a, ζ_a, Ψ_a). The transmissivity of cloudless, but not pure, air varies from about 0.9 when very clean to about 0.6 in haze and smog, with typical values of about 0.84.

The nature and amount of absorption depends on the absorption spectra of the atmospheric gases (Figure 5) and of cloud and other aerosols. Figure 5 demonstrates that the Atmosphere is not a very good absorber of short-wave radiation (0.15 - $3.0\,\mu\mathrm{m}$). Ozone (O_3) is very effective at filtering out ultra-violet radiation at wavelengths less than $0.3\,\mu\mathrm{m}$, and water vapour becomes increasingly important at greater than $0.8\,\mu\mathrm{m}$, but in the intervening band where the intensity of solar radiation is greatest (i.e. near $\lambda_{\max}$) the Atmosphere is relatively transparent. Even the absorption by liquid water drops in cloud is relatively small.

Direct and diffuse irradiance. The portion of the incoming solar radiation that is reflected and scattered, together with that multiply-reflected between the surface and the atmosphere (back-scattered), gives diffuse short-wave radiation (D). As an approximation we may consider this radiant receipt to arrive from all parts of the sky hemisphere, although in cloudless conditions it is greater from the area of the sky around the solar disc and near the horizon. Clouds are very effective at diffusing short-wave radiation.

Finally, the portion of the incoming solar radiation that arrives at the Earth's surface, without being absorbed or diffused, is called the direct-beam short-wave radiation (S). Since it can be approximated as a parallel beam, the irradiance of an exposed surface depends on its orientation to the beam. The total shortwave radiation received at the surface (the *shortwave irradiance* $K_\downarrow$) is:

$$K_\downarrow = S + D \qquad \star \qquad (2.13)$$

In the middle of a clear day the proportion of $K_\downarrow$ arriving as diffuse radiation D is anywhere from 10 to 25% of $K_\downarrow$ depending on the amount of water vapour haze; in smoggy urban and industrial areas it will be even greater. Early and late in the day the diffuse proportion also increases due to the greater path length of the Sun through the Atmosphere.

Transmission geometry. The beam solar radiation on a horizontal surface (S) at Earth's surface is given by :

$$S = K_{Ex} \Psi_a^m \qquad (2.14)$$

The path length (or optical air mass number, m) is expressed as the ratio of the slant path taken by the beam to the zenith distance, so that:

$$m = \sec Z = \frac{1}{\cos Z} \qquad (2.15)$$

This is acceptable for angles of Z less than $80°$ (or β greater than $10°$) but closer to the horizon corrections for refraction and other effects are necessary. It also

[1] http://www.geog.ubc.ca/courses/geob300/applets/sunpath/

applies to locations at sea-level. To account for the effects of altitude values of m should be multiplied by the ratio of the station atmospheric pressure to that at standard sea-level (101.3 kPa). It should be stressed, however, that these will be first approximations. Detailed models which account for the roles of individual atmospheric constituents and their radiative properties are available for more accurate assessment (e.g. MODTRAN, see also ATSC 301).

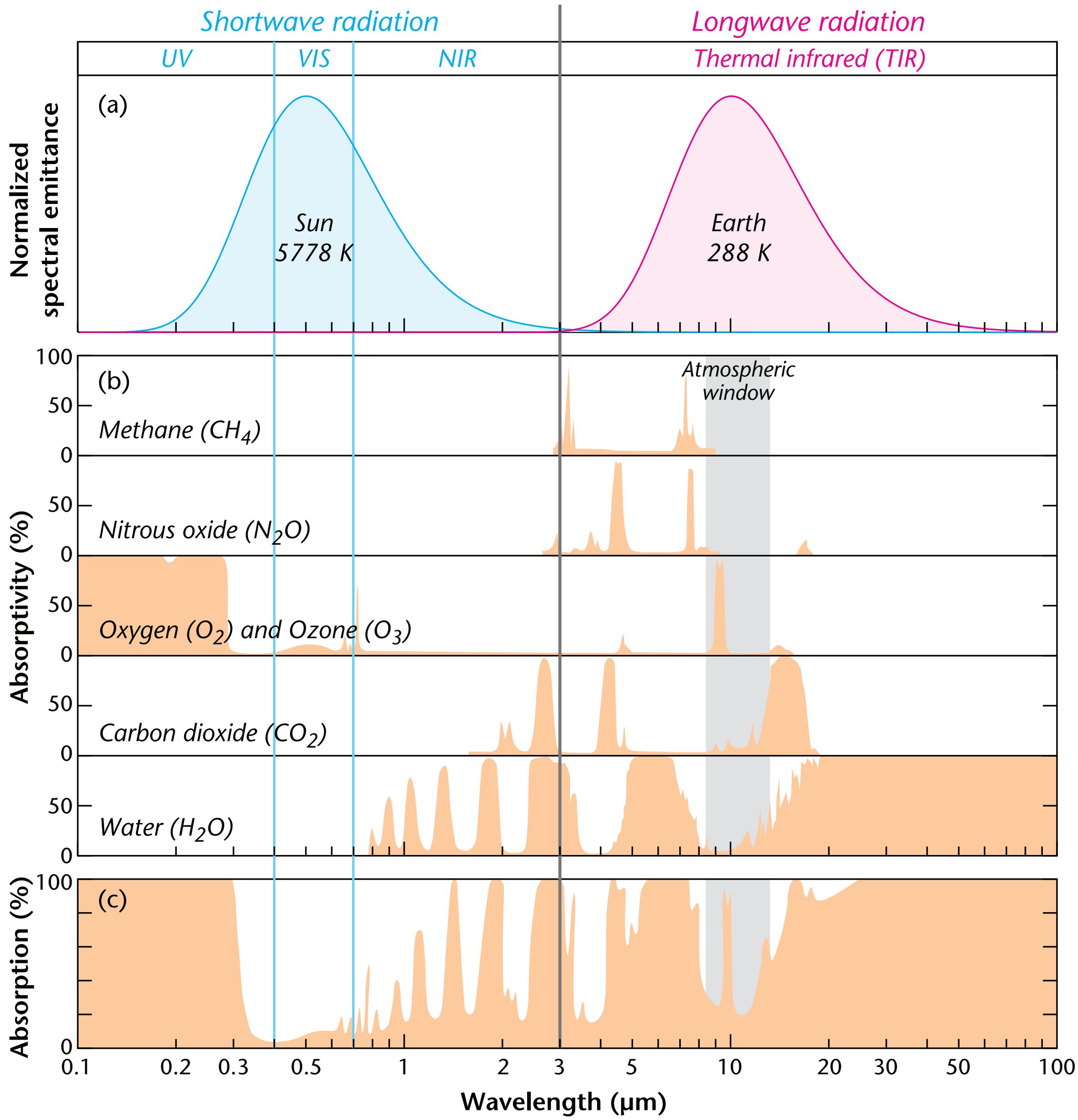

Figure 5: Absorption at various wavelengths by constituents of the Atmosphere, and by the Atmosphere as a whole.

Additional Readings (not required).

Campbell, G. S. and Norman J. M. (2000): 'An Introduction to Environmental Biophysics', Springer, 2nd Ed. Chapter 10, 11.1 - 11.2 (p. 147 - 173)

Arya, S.P. (2001): 'Introduction to Micrometeorology', Academic Press, New York, 2nd Ed. Chapter 3.1-3.3 (p. 28 - 38)